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THE EFFECT OF STAYING POWER ON OFFENSIVE AND DEFENSIVE POWER OF A MODERN WARSHIP

by

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ABSTRACT

This thesis develops the relationship between the three major attributes of a modern warship: staying power, offensive power, and defensive power. Based on W. Hughes' Salvo Model, we show that there is an inverse linear relationship between a ship's staying power and its defensive power and a nonlinear relationship between its staying power and its offensive power. Using the embellishments that E. Hatzopoulos added regarding scouting effectiveness and alertness, we also compare the results for survivability which A. Lalis obtained from the Hatzopoulos model using scouting and alertness. We examine survivability using *defensive power* (or *equivalently staying power*, because of their linear relationship) and also offensive power.

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I. INTRODUCTION

A. BACKGROUND: COMBAT MODELS

The art of naval warfare is complicated primarily because the main actors in naval warfare are human beings. People tend to believe that, "If you want peace, you must be ready for war." Today's electronics technology has only increased the speed at which a battle scenario can change.

A whole science describing the conduct of war has been developed. Accordingly, the existence of aids or tools for the decision-maker has become very important. These aids in understanding are referred to as "Combat Models."

What is a model and what are its features? According to Giordano and Weir, a model is a mathematical construct which is designed to study a particular real-world system or phenomenon. A mathematical model can be a formula, equation or system of equations describing how underlying factors are interrelated [Ref. 1:p. 32]. In other words, "A model is a simplified representation of the entity it imitates or simulates [Ref. 2:p. 1]."

There are two main purposes of a combat model. First, a combat model can be thought of, as previously stated, as the aid or tool used to help the decision-maker. As Hughes says, "A model is useful if a better decision can be made with the information that it adds [Ref. 2:p. 17]."

The more reliable the model, the greater the level of understanding gained and the more sophisticated the decision based on this model will be. One must realize, though, that the models being discussed, called in general "military models," are special types of models. A military model can not consider the thoughts and the feelings of the commander in battle since they are qualitative in nature. One cannot expect that a model, even the best one, will win a battle. What is expected is that a good model will assist a decision-maker in making better decisions more quickly.

The second purpose of a validated combat model is to aid in the interpretation of historical battles. This is of great value to those who believe that by analyzing the battles our predecessors fought we can learn about their mistakes and how to avoid them in the future.

History is the basis for building the future and must always be considered.

> The value of military history is that when analyzed objectively and scientifically, it permits us to project forward the trends of real past experience. This is the only way these relevant lessons of actual combat can be brought to bear on the important national defense issues of today [Ref. 4:p. xxvi].

With a validated model, analysts are better able to study a situation and to determine if a commander made the right decision or if he should have done something different. A model could also describe step-by-step, when and how the opponents came to the critical point which determined victory or defeat.

A computer implementation was not used in this study because the nature of the attributes and their interrelationships is so chaotic that one could draw

conclusions only for a specific set of inputs. As Hughes says, "Computer simulations tend to obscure the essential structure of a modern sea battle, and so mask some important implications for warships' design and the significance of staying power [Ref. 4]."

B. NAVAL COMBAT MODELING

In his monograph entitled "Military Modeling," Hughes distinguishes "descriptive modeling" from "prescriptive and predictive modeling." According to Hughes, the models with which we deal in this study are descriptive models, the purpose of which is to describe and understand the phenomena at hand

The phenomenon in this case is naval combat. As for every model, a naval combat model is designed to make "IF-THEN" statements. In this case, the inputs and the model itself are the "IF." The results, which are the "THEN," follow. Naval combat analysts put particular emphasis on the "IF," because it is not within the power of the analyst to predict everything.

It is useful to add here that the results of both a naval model and a battle depend on the attributes or factors of the unit or platform which is going to fight. Additionally, even if we know the attributes of a unit or force, the attributes of the enemy and his performance in battle are coequal determinants of a naval model's (and a battle's) results.

C. THESIS GOAL AND SCOPE

Based on Hughes' concept of naval warfare, the goal of this thesis is to examine the relationship between the three major elements of a modern warship. These elements, or attributes, are: the staying power; the offensive power; and the defensive power. The objective of this study is to investigate how staying power is related to the offensive and defensive power of a warship.

It is true that most people, sometimes even the designers of a platform, do not pay enough attention to staying power. One reason for this could be that staying power is a kind of hidden element, unlike offensive power and defensive power which are visible, active elements that everyone sees and consequently deals with more extensively. Basically, staying power is an element built into a ship's design. As Hughes says, "Staying power's inherent robustness suggests that it should be treated with great respect [Ref. 4]."

In Chapter II, the necessary and appropriate aspects of combat theory and terminology for this work will be discussed. As combat theory is voluminous, only that which is needed to help the reader better understand this work will be discussed.

In Chapter III, a modern naval model developed by E. Hatzopoulos [Ref. 5] will be described. This is the basic model with which we will work. Also briefly described will be the elaboration and investigation A. Lalis [Ref. 6] did in order to test how sensitive Hatzopoulos' model is to changes of the input parameters of scouting effectiveness and alertness.

In Chapter IV, specific relationships between staying power and defensive power and also between staying power and offensive power will be developed. Investigation will carry out the results from Lalis' work for the two cases of equally and unequally capable ships. The survivability of the warship will be dealt with in terms of staying power and the relationship developed for offensive and defensive power. To do this, alertness will first be shown to be a satisfactory proxy for defensive power (through staying power) and scouting a satisfactory proxy for offensive power.

In Chapter V, conclusions about the lessons learned in this study will be summarized. This chapter will explain why it is useful for a warship to try to increase its staying power. Recommendations for further research and validation of the developed model will conclude this paper.

II. NAVAL COMBAT THEORY

A. INTRODUCTION

In this chapter we summarize the essential ideas and terminology from the existing body of combat theory. This will help the reader to understand the models that we are addressing in the next chapters.

We have limited the discussion of the terms we present in this chapter only to the point of familiarity with essential concepts. The goal of this chapter is to lay the foundation for the development of our model and also to establish the terminology. Most of the concepts analyzed in this chapter are taken from Hughes' work [Ref. 4].

B. COMBAT THEORY

1. Striking or Fighting Power

The striking power of a combat unit is the number of accurate shots fired by it a) per period of time for continuous fire, b) per tightly spaced pulse for salvo fire.

The striking power of a force is the number of accurate shots fired by all units a) per period of time for continuous fire, b) per tightly spaced pulse for salvo fire.

Striking power is diminished for accuracy of fire.

2. Force Strength

Force strength is the number of units which comprise a homogenous force. The total value of a heterogenous force is the weighted sum of the individual unit values measured against a standard unit, e.g., an FFG-7.

3. Staying Power

Staying power is measured by the number of hits that can be absorbed by a unit or force before its combat effectiveness is reduced to zero for the remainder of the engagement.

4. Offensive Power

Offensive power is actually an undefined expression for striking power, fighting power or combat power (defined below) as appropriate to the circumstances.

5. Combat Power

We can say that combat power is the rate of force projection in units of weapon delivered against the enemy by a force. Combat power is a real phenomenon in the sense that the results from a battle are directly related to the quantity of combat power.

Combat power is generated from the combat potential and is also closely related to how the Commander uses his combat potential. Thus, combat

power can be seen as a function of the number of force elements (warships) on the one hand, and the type of forces and rate of their activities on the other.

If P denotes combat power, m denotes the number of elements in a force, and u the rate of the force's actions, the fundamental equation of combat power is given by

$$P = F(m,u)$$

where F is called the command function [Ref. 5]

Another important issue of combat power is the distinction between theoretical and effective combat power. *Theoretical Combat Power* can be roughly considered as the availability of weapons which a unit can deliver per time unit. *Effective Combat Power* is a measurement of the capability of the theoretical combat power to cause damage to the enemy.

For example, if we have the capability to launch ten missiles in a salvo, then our theoretical combat power is ten. If we deliver these ten missiles but only five are aimed so that they can hit the enemy, then our effective combat power is five.

Another way to express combat power is given by Hughes as:

> Combat power is the striking power of a force minus the total hits eliminated by the defensive actions of the target force. Combat power cannot be defined or measured except against a specific enemy force and the actions it takes to diminish the striking power against it.

6. Defensive Power

Defensive power can be expressed as the number of shots (which would hit the unit or force) that will be destroyed or averted by a unit's or force's defenses, that is, the defensive force. Another way to express defensive power is the ability of a unit or force to reduce the "susceptibility" to hits by the enemy.

7. More About Staying Power

As we said earlier, staying power can be seen as the ability of a warship to survive after a hit has been taken from this warship. We can say also that it is the capacity of a warship to absorb the hits fired by the enemy and continue fighting.

Staying power is the warship design element least affected by the particulars of a battle. Staying power has to do much more with the material way that a warship is built, and much less with the other factors that possibly can affect staying power (e.g., crew readiness). Staying power has also to do with the enemy's offensive power and the type of enemy's weapon. For different missiles (explosive weight, type of warhead, etc.) we have a different contribution of our staying power to the reduction of our loss.

In older days, the idea was that staying power (or equivalently survivability) goes up by increasing the size and armor of the warship. The bigger the displacement of the warship, the bigger the staying power and hence, the survivability. As Hughes says:

> The case for staying power in the form of armor ended with the atomic bomb. We could have, we thought, one warship sunk with every hit; survivability would have to come from other means. [Ref. 4]

One way to increase the warship's survivability is, as we will see in Chapter IV, by increasing defensive power by having a high alertness level. Methods to increase staying power are by building warships with material (e.g., special alloys) which reduce the hazard of burning, or with low-observable material that reduces the detection range.

The buoyancy design of the ship, and alternatives in the operational structure, would be some factors which also can affect the designed staying power of the warship. In conclusion, we wish to say that staying power must be thought of not only as a function of displacement but of other design attributes of the warship. To develop the ways to enhance staying power is a matter of detailed engineering design.

8. Combat Work

Combat work is the number of units put out of action (OOA) by a salvo or a period of continuous fire. Combat work may also be the accumulated units put OOA after a series of salvo exchanges. It can be thought of as the ratio of combat power divided by staying power.

9. Alertness (τ): A Proxy for Defensive Power

By full alertness we mean a situation in which the crew of the warship has been warned and is fully ready to confront an attack. For example, we can

say that when a warship is on "general alert" the ship is fully ready to fight and every one is on "battle stations."

We wish to show that the significant effect of alertness (τ) is on defensive power rather than on staying power and more or less than offensive power. Let us examine each warship attribute one at a time.

a. Defensive Power

With high alertness the warship is able to employ its maximum defensive power and thereby to increase its survivability. By having a reduced alertness, first of all, the warship can be surprised and so the defensive system of the warship cannot be employed fast enough. As an example, we can refer to the two cases of the US FFG STARK and HMS FFG SHEFFIELD, in which the two ships had low alertness so that they were surprised by the enemy, their defensive systems did not work, and they were hit by the enemy.

b. Staying Power

When we have high alertness, we also have a high ability to confront the problems which arise in the battle, faster and more effectively. To make clearer what we mean, we will refer here only to the increase we have in the damage control readiness from a high alertness and we will skip the other factors. Damage control is the ability to limit damage caused by enemy hits, such as to repair a failure in the power of the warship, to put out a fire, etc., and in

general to restore the warship to fight again, as strong as possible, after the hits are taken.

By having a high damage control readiness we can reduce the vulnerability of the warship or conversely increase the staying power of the warship. We wish here to give a simple example of what we mean. Assume we have a warship which is designed to absorb two missile hits before it goes out of action. Then this warship has staying power of two missile hits. Also, assume that the warship is hit by a missile and because we assumed the warship was in high alertness, then damage control personnel are able to repair much of the damage caused by the missile. A warship which has taken one hit with staying power of two hits then can absorb one more. A warship in low state of readiness or alertness though, may be out of action after it has received only one.

c. *Offensive Power*

Today, with the automation we have, a warship is able to deliver the maximum amount of offensive power, with only the presence of a small number of operators by simply pushing buttons from the console of a modern combat center. In former days, the number of people at battle stations affected the offensive power more than today.

In conclusion, we can say that alertness is directly related to defensive power, much more than to staying power and it is not greatly related to offensive power. Through defensive power, then, we can increase the survivability of the warship.

10. Scouting Effectiveness (σ): A Proxy for Offensive Power

By scouting we mean the ability of a warship or a force to collect all the important information about the enemy needed to attack effectively. When σ=1, the most effective attack for the minimum cost can be achieved.

As examples of information that could be important for a warship or a force to know about the enemy are the position of the enemy, composition and capability of the enemy force, enemy's intentions and plans, and any other useful combat information.

The correct use of scouting effectiveness by the Commander can be decisive for the results of the combat. A scouting advantage is necessary if we want to surprise the enemy and a) avoid the enemy's defensive power, or b) attack before the enemy can launch his attack.

It is understood that full scouting effectiveness ($\sigma = 1$) is necessary and sufficient if we want to develop the full value of our offensive power. If we know exactly where each enemy unit is, for example, then we are sure where to "send" our missiles. Regarding this point we like to say that as our scouting effectiveness increases, our ability to fire more precisely increases and so does our firepower, but not our offensive potential (the number of missiles we are able to fire remains the same). Therefore, we can say that having full scouting effectiveness weakens the enemy to the extent of our full firepower. That is, our full offensive power is the minimum required.

Scouting effectiveness only contributes to survivability by increasing the damage to an enemy before he attacks. Thus, we conclude that scouting has more to do with the offensive power (in the way we discussed) than with the other attributes. The main effect of a high force scouting is to maintain full offensive power rather than defensive or staying power.

11. Uncertainty in Naval Combat

This is to emphasize the particularity of the naval combat models comparing them with other kinds of models. "Uncertainty is inherent in combat." [Ref. 3] We can define uncertainty as the situation in which we are not sure that we can control the different states (or steps) of a combat and especially the results of the combat. Uncertainty increases whenever we have unclear or incomplete information about the enemy. To this point we would like to add that the uncertainty is even greater when we do not know the situation or the state of our force. Even if we know the exact present state of the enemy, uncertainty still exists because we do not know how the enemy will act in the future.

The main reason for uncertainty in naval combat is because human beings are involved in the combat process and the human beings are, most of the time, the most uncontrolled variables. This is historically proven in many cases. As an example, we would like to refer to the famous decisive naval battle of Salamis in 480 B.C. between Greeks and Persians. The Persian fleet was composed of eight hundred ships with enormous offensive and defensive power. The Greek

fleet was composed of only three hundred and seventy ships. The prediction was that it was impossible for the Greeks to win.

The Greek admirals laid a trap. They guided the Persian fleet into the straits where it was very difficult for them to maneuver their bigger ships. The result was catastrophic for the Persians. If the Persian king Xerxes had the uncertainty factor in his mind, he would not have fallen into the trap. He was too sure of his victory. What happened was that with the tactical advantage of the terrain the Greek fleet diminished the combat power of the Persian fleet.

C. THE MEANING AND VALUE OF THE NUMBERS OF COMBAT UNITS

One of the most important parameters in a combat is the number of units each side has. For example, if the units on side A are twice as numerous as the units on side B, it can be shown that then each unit on side B will need to have twice the offensive power, defensive power, and staying power of side A in order to have parity [Ref. 4]. The value of numbers was and is always a major factor that we have to think about.

To show how important numbers are, let us present one of the first combat models developed and which is still a very important model. We are talking about Lanchester's Model. Let x and y be two forces, and also let $x(t)$ and $y(t)$ be the sizes of these forces at time t, with a and b as attrition rate coefficients for the x and y forces, respectively.

1. Square Law

The following equations are known as the Lanchester's square law and are used to represent a situation where *attrition* to each side is *proportional to the number of units remaining on the other*. These two equations are:

$$\frac{dx(t)}{dt} = -a \cdot y(t) \tag{2.1}$$

$$\frac{dy(t)}{dt} = -b \cdot x(t) \tag{2.2}$$

where the attrition coefficient a represents the quantity of casualties of x per fire of y in each time unit and correspondingly for attrition coefficient b.

Now, if we let x_o and y_o be the initial sizes of x and y forces at the beginning of the engagement, and x_f, y_f the final numbers of survivors on each side, then solving equations (2.1) and (2.2) we have the following state equation:

$$b(x_o^2 - x_f^2) = a(y_o^2 - y_f^2) \tag{2.3}$$

Equation (2.3) can be used to determine the necessary and sufficient condition for a force to win. Now, if we let x_f=0 or y_f=0 (which means that battle is continued to its finish) then we can say that

$$force\ x\ wins\ if\ \frac{x_o}{y_o} > \sqrt{\frac{a}{b}}$$

and

$$force\ y\ wins\ if\ \frac{x_o}{y_o} < \sqrt{\frac{a}{b}}$$

To determine the size of x force at time t, we differentiate equation (2.1) substitute to equation (2.2) and so we obtain the following second-order differential equation:

$$\frac{d^2x}{dt^2} - bax = 0 \qquad (2.4)$$

Solving equation (2.4) we have the size of force x at time t:

$$x(t) = \frac{1}{2}\left(x_o - \frac{\sqrt{a}}{b} \cdot y_o\right) e^{\sqrt{ab} \cdot t} + \frac{1}{2}\left(x_o + \frac{\sqrt{a}}{b} \cdot y_o\right) e^{-\sqrt{ab} \cdot t} \qquad (2.5)$$

To determine how long the battle will last, we assume that force y wins. Then x(t) has to be zero in equation (2.5) and solving for t we have the time a battle will last:

$$t = \frac{1}{2\sqrt{ab}} \ln \left[\frac{\left(1 + \frac{x_o}{y_o}\sqrt{\frac{a}{b}}\right)}{\left(1 - \frac{x_o}{y_o}\sqrt{\frac{a}{b}}\right)} \right] \tag{2.6}$$

What we can see from equations (2.3) to (2.6) is that the number of combat units are very important for the result of the battle. For that reason, in the remainder of our work we will talk only about equal number of forces in order to avoid the effects of numerical advantage and the value of number on each side. We will conduct the analysis with one unit (or equal numbers) since it is already clearly understood that the more numerous a force is, the stronger the force will be.

III. BASIC NAVAL MODELS AND ELABORATIONS

A. THE BASIC NAVAL SALVO MODEL

This chapter summarizes the Basic Naval Salvo Model derived from Chapter VII of Hughes' work, *The Value of Warship Attributes*, September 1992. The Salvo model is the basis of our investigation of the value of staying power with respect to other attributes.

1. Model Assumptions

- The striking power of the attacker is the number of accurate (good) shots launched.
- Good shots are spread equally over all targets. A uniform distribution is not necessarily the best distribution. If each target's defense extracts an equal number of accurate shots, the whole strike may be defeated, whereas an uneven distribution concentrated against only some targets would put at least those targets out of action.
- Counterfire by the target force eliminates with no "leakage" all good shots until the force defenses are saturated, after which all good shots are hits. Mathematically, a subtractive process best describes the effect of counterfire.
- Hits on a target force will diminish its whole fighting strength linearly and proportionate to the remaining hits the target force can take before it is completely out of action.
- Weapon range is "sufficient" on both sides. In other words, neither side has a weapon range and scouting advantage such that is can detect, tract and target the other while standing safely outside the range of the enemy's weapons.

2. Force-On-Force Equations

The combat work achieved by a single salvo fired by a force A is:

$$\Delta B = \frac{\alpha \cdot A - b_3 \cdot B}{b_1} \tag{3.1}$$

and by a force B:

$$\Delta A = \frac{\beta \cdot B - a_3 \cdot A}{a_1} \tag{3.2}$$

where in equation (3.1)

ΔB is the number of ships lost in force B caused by force A's salvo

α is the offensive power of a single unit in force A

b_3 is the defensive power of a single unit in force B

b_1 is the staying power of a single unit in force B

$\alpha \cdot A$ is the total offensive power of force A

$b_3 \cdot B$ is the total defensive power of force B.

The corresponding terminology holds for equation (3.2), also. The combat power of a salvo is measured in hits that damage the target force, and is the numerator of the equations (3.1) and (3.2), respectively. Dividing total salvo combat power by the number of hits a target can take before it is out of action (staying power), the result is the enemy ships out of action.

3. Model-Based Conclusions

- Next we need to see what fraction of each force can be put out of action after a salvo exchange. This is expressed by the following equations:

$$\frac{\Delta B}{B} = \frac{\alpha \cdot A - b_3 \cdot B}{b_1 \cdot B} \tag{3.3}$$

$$\frac{\Delta A}{A} = \frac{b \cdot B - a_3 \cdot A}{a_1 \cdot A} \tag{3.4}$$

Now if we want to have a comparative effectiveness of the two sides, we divide equation (3.3) by equation (3.4) to obtain what we call the Fractional Exchange Ratio (FER):

$$FER = \frac{\Delta B / B}{\Delta A / A} = \frac{(\alpha \cdot A - b_3 \cdot B)(a_1 \cdot A)}{(\beta \cdot B - a_3 \cdot A)(b_1 \cdot B)} \tag{3.5}$$

When FER > 1 then force A will have forces remaining when all of force B is out of action. When FER < 1 then force B will have forces remaining when all of force A is out of action.

- Excessive offensive and defensive power in the form of overkill will have a significant effect on the results.
- The Fractional Exchange Ratio is unreliable when an overkill situation exists.
- In general, instability is great when force combat power (numerator in equation (3.1) and (3.2)) is large in comparison with force staying power (denominator in equation (3.1) and (3.2)). If unit staying power a_1 or b_1, cannot easily and affordably be added, then force staying power can only be increased and stability restored by increasing the number of units in force A or B.

4. Discussion

Equations (3.1), (3.3) and (3.5) are the basic forms for exploratory analysis. The unstable circumstance of a very strong combat power on both sides relative to their staying power argues under all circumstances in favor of delivering unanswered strikes. The apparent instability and chaotic behavior of the basic salvo model implies the limited value of studies using specific scenarios and ship characteristics in any detail until the general nature of warship attributes and their interrelationship is understood.

5. The Advantage of Numbers

Since we have introduced the Naval Salvo Model, we wish again to draw attention to one of Hughes' most important conclusions about the importance of numbers.

> The attribute that is the most consistently advantageous in force-on-force engagements is the number of combat units. For example, if forces engaged on side A are twice as numerous as side B, then for combat parity, each unit of side B must have twice the striking power, twice the staying power, and twice the defensive power of each side of side A's combat units. The special combat advantage of numbers seems to apply under a very wide set of circumstances. [Ref. 4:pp. 38-39]

From equation (3.5) we have:

$$FER = \frac{\Delta B/B}{\Delta A/A} = \frac{(\alpha\cdot A - b_3\cdot B)(a_1\cdot A)}{(\beta\cdot B - a_3 A)(b_1\cdot B)}$$

To have parity we need $FER = \dfrac{\Delta B/B}{\Delta A/A} = 1.$

a. Let A=2B

Then FER becomes:

$$FER = \frac{(2\alpha B - b_3\cdot B)(2a_1\cdot B)}{\beta\cdot B - 2a_3 B)(b_1\cdot B)} = \frac{(2\alpha - b_3)(2d_1)}{(\beta - 2a_3)(b_1)}$$

b. Let $\beta=2\alpha$, $b_3=2a_3$ and $b_1=2a_1$

Then substituting these values to the above equation we have:

$$\frac{(\beta - b_3)(b_1)}{(\beta - b_3)(b_1)} = 1.$$

What we can conclude from the above is the dominant advantage of numbers, and we no long need to analyze the worth of this attribute. Of course, instead of doubling the value of *each* attribute of the ships on the side with half the numbers, other combinations of offensive, defensive, and staying power are possible to give parity. What might be such a combination instead of simply doubling each attribute? For the remainder of the thesis we will examine forces

of equal numbers, and the relative advantage of their individual attributes, namely offensive, defensive, and staying power.

B. ELABORATIONS AND INVESTIGATIONS

In 1990, Hatzopoulos, using the Basic Salvo Model, explored how to deal with human factors that affect the outcome of a naval battle. [Ref. 5] The human factors he used are: scouting effectiveness, alertness, leadership, morale, and training. He concluded that while the value of human factors will always be difficult to quantify, the manner in which they will affect salvo warfare outcomes is easy to see and represent. The embellishment Hatzopoulos added to the Basic Salvo Model was to introduce the "human factors" to each equation. In his model, Hatzopoulos assumed the following:

- Scouting effectiveness, σ_A or σ_B, takes values between 0 and 1 that measure the extent to which striking power is diminished due to less than perfect targeting and distribution of fire against the target force.
- Similarly, defender alertness, τ_A or τ_B, takes values between 0 and 1 that measure the extent to which the defensive is diminished due to less than perfect readiness or fire control designation to destroy the missiles of an enemy attack.

We will not go further with Hatzopoulos' other human factors affects. For purposes of our study, we only need to write the Basic Salvo Model with these two human factors.

The modified Hatzopoulos model is as follows:

$$fractional\ loss\ of\ A \equiv \frac{\Delta A}{A} = \frac{\sigma_B \cdot \beta \cdot B - \tau_A \cdot a_3 \cdot A}{a_1 \cdot A} \tag{3.6}$$

and

$$fractional\ loss\ of\ B \equiv \frac{\Delta B}{B} = \frac{\sigma_A \cdot \alpha \cdot A - \tau_B \cdot b_3 \cdot B}{b_1 \cdot B} \tag{3.7}$$

where

σ_A is the scouting effectiveness of each unit of force A

$\sigma_A \cdot A$ is the total scouting effectiveness of force A

σ_B is the scouting effectiveness of each unit of force B

$\sigma_B \cdot B$ is the total scouting effectiveness of force B

τ_A is the alertness of each unit of force A

$\tau_A \cdot A$ is the total alertness of force A

τ_B is the alertness of each unit of force B

$\tau_B \cdot B$ is the total alertness of force B.

In 1991, A. Lalis initiated the validation of Hatzopoulos' Naval Combat Model by carrying out an extensive sensitivity analysis. [Ref. 5] This validation proceeded in two steps. First, the model was used to analyze the hypothetical data from missile and pulse weapon naval battles. The results of the model were then examined from the standpoint of "reasonableness." Second, sensitivity analyses were carried out to determine how sensitive the model's output is to

changes in the inputs for the two model parameters, scouting effectiveness and alertness.

Lalis' general approach was to analyze the sensitivity of Hatzopoulos' model through the use of two ratios. The first was a ratio of one force's remaining staying power to that of the other force, after a salvo exchange. The second was to apply the Fractional Exchange Ratio which as we have seen represents the proportion of each force that has been lost after an exchange of missiles.

From Lalis' work, we will focus on the results he found using the Fractional Exchange Ratio. More specifically, and for the reasons we have explained in Chapter II and III regarding the meaning of the numbers of combat units in each force, we will concentrate our attention on the cases of numerically equal forces.

There are two general scenarios Lalis used to validate Hatzopoulos' model. The first is when we have equally capable forces. Both forces have the same unit offensive power and defensive power. The second is when we have unequally capable forces. Each unit of one force (for convenience, force B) has greater offensive and defensive power than the other force (force A).

We show below now the results Lalis obtained regarding which factor (scouting effectiveness or alertness) is most sensitive to incremental changes.

TABLE I. MODEL SENSITIVITY WHEN OFFENSIVE AND DEFENSIVE POWER ARE EQUAL ($\alpha = \beta$, $a_3 = b_3$)

Variation	Force's A Factor for which Model is Most Sensitive	Force's B Factor for which Model is Most Sensitive
Equality of Number (A=B)	σ_A	σ_B

TABLE II. MODEL SENSITIVITY WHEN FORCE B'S OFFENSIVE AND DEFENSIVE POWER ARE GREATER THAN FORCE A'S ($\beta > \alpha$, $b_3 > a_3$)

Variation	Force's A Factor for which Model is Most Sensitive	Force's B Factor for which Model is Most Sensitive
Equality of Number (A=B)	τ_A	τ_B except σ_B if σ_A is high

In Chapter IV, using Hatzopoulos' model, we will conduct our own sensitivity analysis of offensive and defensive power by using their proxies of scouting effectiveness and alertness, respectively (as we have seen in Chapter II). Then we will compare the results of Lalis' work with the results we obtained through our approach.

IV. THE VALUE OF STAYING POWER IN MODERN WARSHIPS

The most interesting results in naval combat are the losses suffered on each side after a salvo has been fired, and the remaining staying power and the combat power on each side. The primary goal of this study is to show how and why the Commander is concerned about the staying power of his ship or his force.

Staying power has been defined as the number of hits a platform can absorb before suffering a firepower kill. Staying power of a force can be spoken of in the same way as single unit staying power. The aggregate staying power of a force can be considered to be the sum of the total number of hits all single units can absorb and still be able to fight. This study will now investigate the relationship between staying power on one hand, and defensive and offensive power on the other.

Granted that staying power can, in most respects, only be increased by a new design of the unit, the Commander has to try to find other ways to increase the survivability of the unit (e.g., by alertness, countermeasures, etc.). We do not claim that increasing a warship's ability to absorb hits is equal in value to an increased effectiveness in our defensive systems or combat potential (i.e., firing more missiles). What is gained by increasing staying power is that, a) regarding defensive power, more missiles can be put out of action, which is equal to a

greater ability to defend, b) regarding offensive power, there is more security in developing maximum firepower.

A. MODEL DEVELOPMENT

As seen in Chapter III, equations (3.3) and (3.4), the fractional loss of a force A is given by:

$$\frac{\Delta A}{A} = \frac{\beta \cdot B - a_3 \cdot A}{a_1 \cdot A} \tag{4.1}$$

and the loss of a force B is given by:

$$\frac{\Delta B}{B} = \frac{\alpha \cdot A - b_3 \cdot B}{b_1 \cdot B} \tag{4.2}$$

where a_1 = staying power of a single unit of force A

$a_1 \cdot A$ = total staying power of force A

b_1= staying power of a single unit of force B

$b_1 \cdot B$ = total staying power of force B

α = offensive power of a single unit of force A

$\alpha \cdot A$ = total offensive power of force A

β = offensive power of a single unit of force B

$\beta \cdot B$ = total offensive power of force B

a_3 = defensive power of a single unit of force A

$a_3 \cdot A$ = total defensive power of force A

b_3 = defensive power of a single unit of force B

$b_3 \cdot B$ = total defensive power of force B

As explained in Chapter III, the number on each side is the most important attribute for improving results of the battle. For this reason and for the purpose of development and exposition we set aside further considerations of numerical inequality and henceforth assume equal numbers of units of forces A and B.

Now, if A=B and equation (4.1) is divided by equation (4.2), the result is:

$$\frac{\Delta A}{\Delta B} = \frac{\dfrac{\beta - a_3}{a_1}}{\dfrac{\alpha - b_3}{b_1}} = \frac{\beta - a_3}{\alpha - b_3} \cdot \frac{b_1}{a_1} \qquad (4.3)$$

Equation (4.3), as explained in chapter three, is called Fractional Exchange Ratio (FER) and it is a robust basis of comparison, or Measure of Effectiveness (MOE) where the effectiveness now is the fractional loss on each side.

Thus, if:

$\frac{\Delta A}{\Delta B} < 1$, then force A wins

$\frac{\Delta A}{\Delta B} > 1$, then force B wins

$\frac{\Delta A}{\Delta B} = 1$, then the battle is a stalemate

B. STAYING POWER AND DEFENSIVE POWER

1. Development of the Relationship

As has been shown, staying power is the ability of a ship to absorb hits before suffering a firepower kill and it is measured by the number of hits. Defensive power is the ability of a platform to reduce its susceptibility to hits by the enemy. It is measured by the number of enemy missiles that would hit which are shot down by the ship.

Using symmetry, we will develop the relationship for force A (corresponding results hold true for force B). Using equation (4.3), we solve for defensive power a_3 and we have:

$$a_3 = \beta - \frac{\Delta A}{\Delta B} \cdot \frac{(\alpha - b_3)}{b_1} \cdot a_1 \tag{4.4}$$

Equation (4.4) gives the relationship between defensive power a_3 and staying power a_1. Since $\Delta B = \frac{\alpha - b_3}{b_1}$ equation (4.4) is equal to

$$a_3 = \beta - \Delta A \cdot a_1 \tag{4.5}$$

Equation (4.5) is a decreasing linear model. The model is defined when

$$0 \le a_1 \le \frac{\beta}{\Delta A}$$

and is undefined elsewhere. A typical sketch of the model is shown in Figure 1.

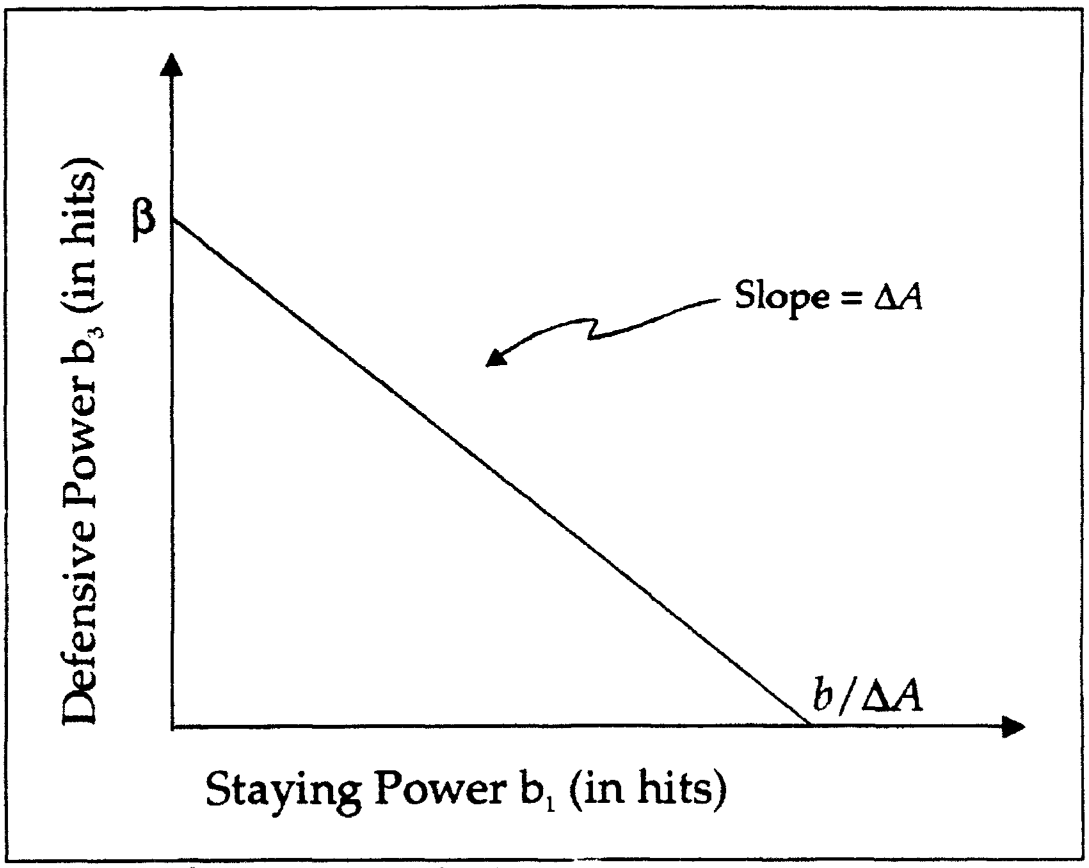

Figure 1. Defensive Power v. Staying Power

What is seen from equation (4.5) and Figure 1 is that for:

$$a_1 = 0,\ then\ a_3 = \beta$$
$$a_1 = \frac{\beta}{\Delta A},\ then\ a_3 = 0.$$

The meaning of this relationship (equation (4.5)) to the Commander is that for a given defensive power a_3 and a given staying power a_1 there is a loss ΔA. If there is an increase in staying power a_1, then one can see that there can be the same amount of loss ΔA, but now using smaller defensive power a_3.

By increasing staying power a_1, a unit is able to absorb more missiles. We see from equation (4.5) that if force A has a small loss, ΔA, then by increasing its staying power a_1, there is a small savings in defensive power a_3 required to maintain the original value of ΔA. This is so because the slope (ΔA) in equation (4.5) is small.

This is consistent with reality. By having small loss, ΔA, the unit already has a defensive power strong enough to shoot down the enemy's missiles. In this case, there is no need to increase staying power a_1 since there is a strong defensive power a_3. If a unit has greater staying power a_1, then the gain achieved in the defensive power a_3 is small and that increment contributes very little to reducing the amount of defensive power a_3 needed.

In the case of a big loss ΔA being suffered (up to $\Delta A = a_1$), then by increasing the staying power a_1, there is a big gain in defensive power a_3. This is

so because the slope (ΔA) in the model represented by equation (4.5) is large (steep).

This is true because after facing a big loss, ΔA, a unit is no longer able to defend (to shoot down the enemy's missiles) enough. By increasing our staying power a_1, the gain achieved in reducing the amount of defensive power a_3 is significant.

Let's examine some cases that may happen in a battle. Suppose that our warship has no staying power but is able to shoot down all the enemy's missiles ($a_3=\beta$). Then our unit does not have to worry about the enemy, since it is able to survive by simply defending. For example, there is no need for our unit to be armored to absorb more hits, since none of the enemy's missiles will hit our unit.

Now suppose that our warship is more armored than before, or equivalently has bigger staying power. In the first case, the survivability of our warship depends entirely on our ability to defend against the enemy's missiles. If something, like a failure in the defending system, happens and our unit is hit by only one missile, then the unit's fighting power reduces to zero.

In the second case, where we have a bigger staying power, if something unexpected happens and we are hit, we still can hope to survive, and win, because of our ability to absorb a hit and still be able to fight.

Considering (for our example) that our goal is to survive to continue fighting, we can do this either by having a high defensive power and a low

staying power or by having a lower defensive power and a higher staying power. But suppose that our warship is surprised by the enemy. In this case, it is very difficult for us to defend because we're not ready. More of the enemy's missiles will hit us. In the case where a missile is coming and we were not able to destroy it using a defensive action, then the only thing that can save us is to possess the ability to absorb the hit and continue fighting. We conclude that it is desirable to have high staying power to offset a weakness caused by the surprise.

2. Discussion

From equation (4.5) when A=B we have

$$a_1 = \frac{\beta - a_3}{\Delta A} \tag{4.7}$$

Now, if we increase staying power a_1 and keep β and ΔA constant then our defensive power is getting smaller.

This means that having a very big staying power a_1 we can have a very small defensive power a_3 and still have the same ΔA. The opposite is true also. If we have a very small staying power a_1 we must have a very big defensive power a_3 if we want at least to have the same ΔA.

C. STAYING POWER AND OFFENSIVE POWER

1. Development of the Relationship

The offensive power or striking power as defined on page 6 is the number of accurate shots fired by a platform that would hit their targets without

the presence of defensive actions. It is measured as we have seen by the number of accurate shots a platform can fire in a salvo. As we did with the defensive power, we are going to develop the relationship between staying power and offensive power for force A. Using equation (4.3) we must solve for offensive power α (for equal number A and B of opposite forces):

$$\alpha = \frac{\Delta B}{\Delta A}\cdot\frac{(\beta - a_3)}{a_1}\cdot b_1 + b_3 \tag{4.8}$$

Equation (4.8) gives the relationship between the offensive power α and staying power a_1. If we put equation (4.8) in a more general form we have:

$$\alpha = \frac{k}{a_1} + b_3$$
$$where\ k = \frac{\Delta B\cdot(\beta - a_3)}{\Delta A}\cdot b_1$$

which is a hyperbolic curve.

So offensive power α and staying power a_1 are related in a non-linear "hyperbolic" relationship. A typical sketch of this relationship is given in Figure 2.

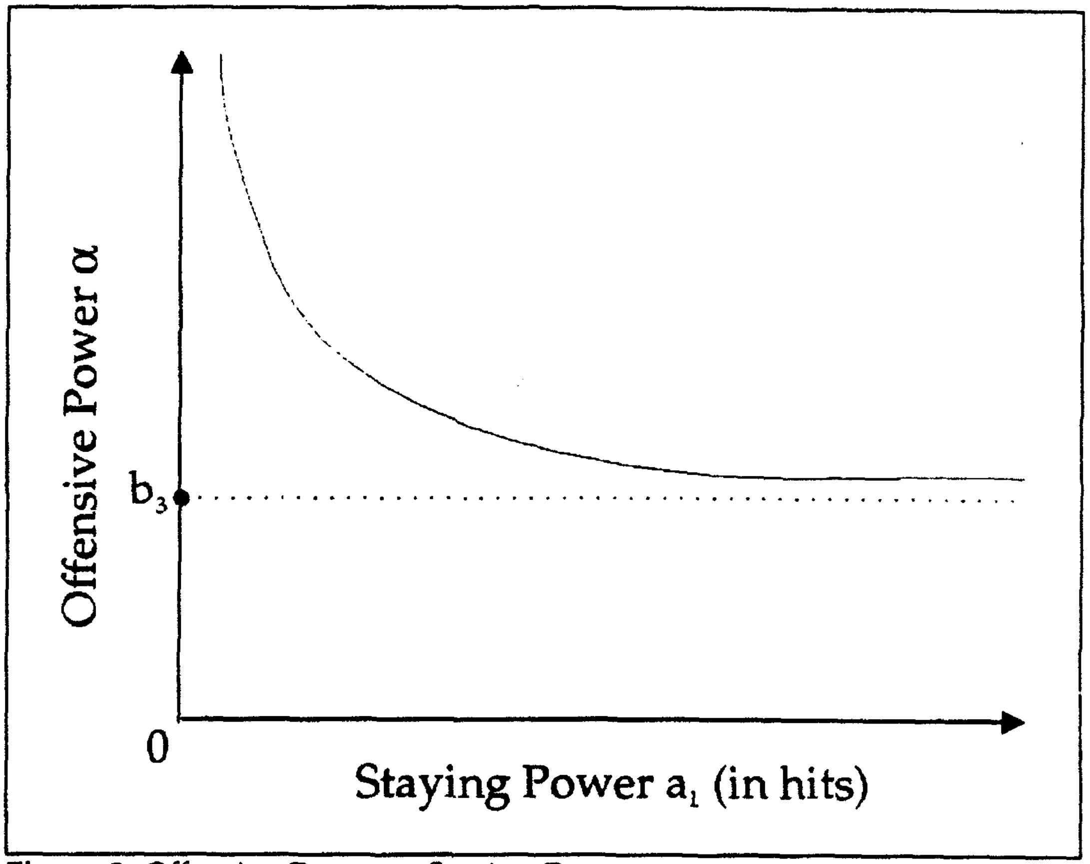

Figure 2. Offensive Power v. Staying Power

What we see from equation (4.8) and Figure 2 is that for

$$a_1 = 0, \textit{ then } \alpha = \infty$$
$$a_1 = \infty, \textit{ then } \alpha = b_3$$

and what we can say is that it is an unbounded relationship.

What a Commander does with the offensive power α and the staying power a_1 is the following: Given an offensive power α, and a staying power a_1, we have a loss ΔA and a loss ΔB. If we increase the staying power a_1, then we can have the same loss ratio $\frac{\Delta B}{\Delta A}$ with a smaller amount of offensive power α.

It is true that if we increase the staying power a_1 (or the ability for the platform to absorb more hits) this does not directly increase the ability to deliver more hits (more missiles). However, by increasing the staying power a_1, the "benefit" we get in offensive power α is to increase our "safety" while we deliver our offensive power α.

The reason why the staying power a_1 and the offensive power α follow a hyperbolic curve is the following: when we deliver our offensive power α, we want to have a strong staying power a_1 in order to avoid a big loss from the enemy's hits, and thereby maintain a winning FER, $\frac{\Delta B}{\Delta A}$. After we already have the "safety" we need to be sure that we will not have problems from the enemy's hits, then further increasing the staying power a_1 does not add much to the offensive power α. For this reason the curve converges asymptotically to the value of the enemy's defensive power b_3.

To explain better why the relationship between the offensive power α and staying power a_1 is more closely related to defensive power a_3 than to offensive power α, let us give a very simple example. Imagine that we have a strong gun which is sited in a very sensitive tower. There are many persons to fire the gun by only one to protect the sensitive tower. With this scenario, it is difficult for the gun to fire more than one time if the one person who is to protect the tower is not able to do so. This is true because we see how easy it is for the enemy to destroy the tower with the strong gun after the gun fires only one salvo. Now we understand how the staying power of the tower affects survivability directly and thereby sustains the offensive power of the gun indirectly.

2. Discussion

From equation (4.8) if the staying power a_1 is very big (theoretically infinite) then the offensive power α is approaching the enemy's defensive power (in our case, $\alpha = b_3$). The meaning of this is that no matter how big our staying power a_1, we must still have an offensive power α big enough to overcome the enemy's defensive power b_3. If the staying power a_1 is very small (theoretically zero), then the offensive power α must be very big (theoretically infinite) in order to maintain parity in the loss ratio $\frac{\Delta B}{\Delta A}$.

The meaning of this is that we cannot absorb even one of the enemy's hits. In this case, if we want to survive we have to either a) attack first and using sufficient offensive power α to eliminate the enemy, or b) our defensive power a_3 must equal enemy's offensive power β.

At this point it would be helpful to examine the concepts and equations developed so far with some simple numerical examples.

Example 1: Staying Power and Defensive Power. Assume that both forces A and B consist of the same number of ships so we can think only about one ship in each force with attributes α, a_3, a_1 and β, b_3, b_1, respectively. The characteristics which are arbitrarily chosen are given in the Table III. The results for one ship on each side will be the same for N ships on each side.

TABLE III. CHARACTERISTICS OF EACH UNIT

Attribute	Force A	Force B
Staying Power	$a_1 = 2$	$b_1 = 1$
Defensive Power	$a_3 = 3$	$b_3 = 3$
Offensive Power	$\alpha = 4$	$\beta = 4$

For equation (4.1) we have $\Delta A = \dfrac{\beta - a_3}{a_1} = \dfrac{4-3}{2} = 0.5$, or each ship of force A suffers a 50% loss.

From equation (4.2) we have $\Delta B = \dfrac{\alpha - b_3}{b_1} = \dfrac{4-3}{1} = 1$ or each ship of force B suffers a 100% loss.

Now, let's say that we are a ship of force A and see what happens to our defensive power a_3 if we change our staying power a_1 to some other representative values. We will keep all the other attributes constant.

Using equation (4.5) we have:

for a_1 = .5 we get a_3 = 3.75

for a_1 = 1 we get a_3 = 3.5 (we halve staying power)

for a_1 = 2 we get a_3 = 3

for a_1 = 3 we get a_3 = 2.5

for a_1 = 4 we get a_3 = 2 (we double staying power)

for a_1 = 5 we get a_3 = 1.5

To give an explanation, we see that in the second set of data we have halved the staying power from the value of two (2) to the value of one (1), then in order to keep the same loss ΔA we need to *increase* our defensive power from three (3) to three and a half (3.5). In the fifth set of data we have doubled the staying power from two (2) to four (4), we may *decrease* our defensive power from three to two (2) and still have the same loss ΔA.

A graphical presentation of the results is given in Figure 3.

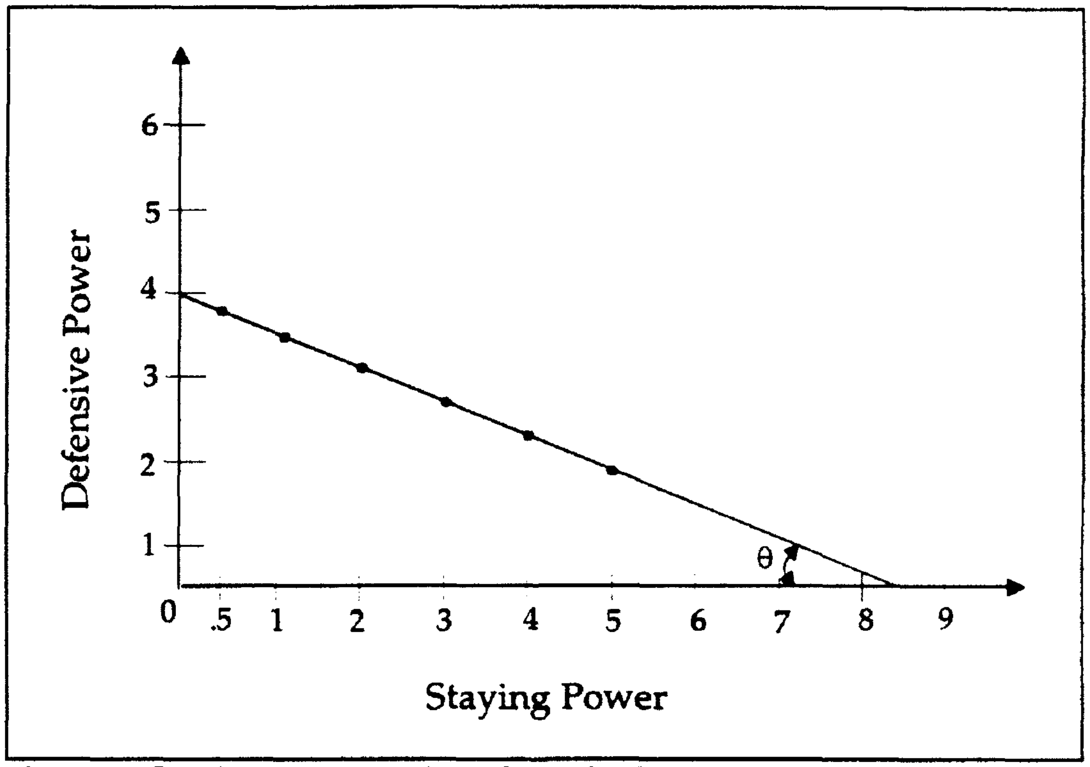

Figure 3. Graphical Presentation of Results from Example 1

Of course, if we increase our staying power and do not decrease our defensive power, then we have "benefit" in our loss ΔA which is smaller.

If we think of the defensive power as a dependent variable and staying power as an independent variable, then the slope of the inverse linear model is equal to ΔA and the angle θ is equal to arc tan (ΔA) = arc tan $(0.5) = 26.56°$

Example 2: Staying Power and Offensive Power. The same assumptions as in Example 1 will be made. The characteristics of each unit of force A and B are given in Table IV.

TABLE IV. CHARACTERISTICS OF EACH UNIT

Attribute	Force A	Force B
Staying Power	$a_1 = 2$	$b_1 = 1$
Defensive Power	$a_3 = 3$	$b_3 = 3$
Offensive Power	$\alpha = 4$	$\beta = 4$

Using equations (4.1) and (4.2) we again have

$$\Delta A = \frac{\beta - a_3}{a_1} = \frac{4-3}{2} = 0.5$$

or the loss of each ship of force A is 50%,

$$\Delta B = \frac{\alpha - b_3}{b_1} = \frac{4-2}{2} = 1$$

or the loss of each ship of force B is 100%.

Again, let us be force A. Using equation (4.8) we will give some representative values of staying power and we will see how the offensive power changes while we keep $\Delta B/\Delta A$ unchanged.

for $a_1 = 1$ we get $\alpha = 6$ (we halve staying power)

for $a_1 = 2$ we get $\alpha = 4$

for $a_1 = 3$ we get $\alpha = 3.333$

for $a_1 = 4$ we get $\alpha = 3$ (we double staying power)

for $a_1 = 5$ we get $\alpha = 2.8$

for $a_1 = 6$ we get $\alpha = 2.666$

In the first case we halved the staying power from two (2) to one (1). In order to have the same $\frac{\Delta B}{\Delta A}$ we need to increase our offensive power from four (4) to six (6). In the fourth case we double the staying power from two (2) to four (4). In order, again, to have the same loss ratio $\left(\frac{\Delta A}{\Delta B}\right)$ we are able to decrease our offensive power from four (4) to three (3). Now in the case we increase our staying power and we keep the same offensive power, then the benefit we get is in the reduction of our loss (ΔA). A graphical presentation of the results is given in Figure 4.

If we think of the offensive power as the dependent variable and the staying power as the independent variable, we see that we have a hyperbolic curve which converges asymptotically to the enemy's defensive power $b_3 = 2$.

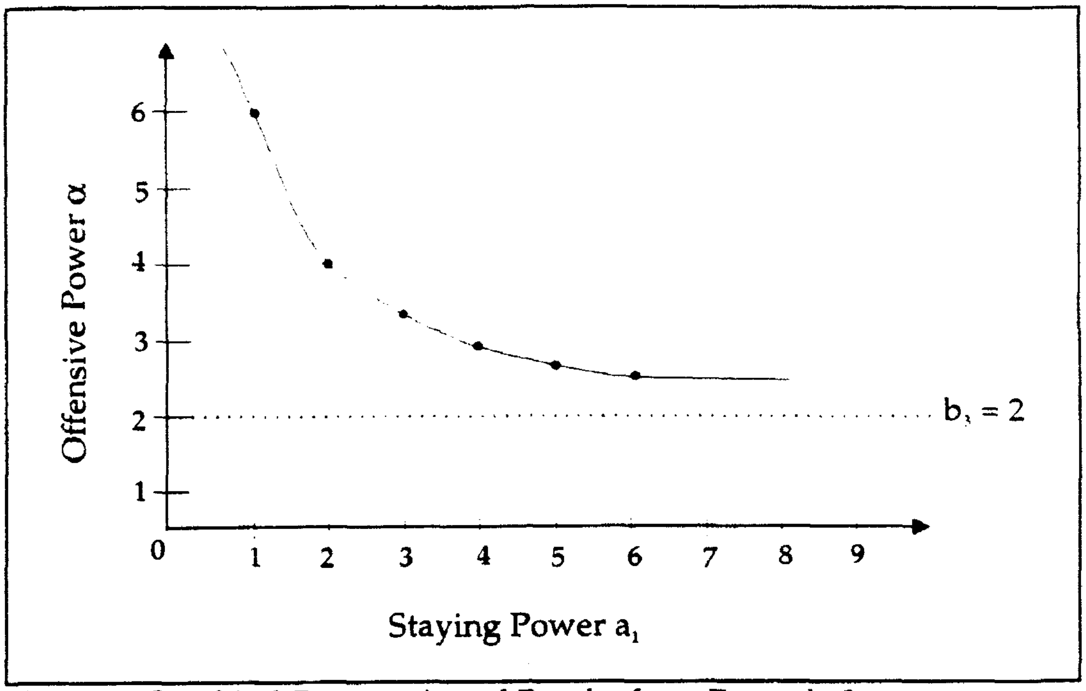

Figure 4. Graphical Presentation of Results from Example 2

D. SURVIVABILITY THROUGH DEFENSIVE AND OFFENSIVE POWER

In order to compare the conclusions by Lalis, displayed in Tables I and II, page 25, we use Hatzopoulos' equation for loss of force (A):

$$\frac{\Delta A}{A} = \frac{\sigma_B \cdot \beta \cdot B - \tau_A \cdot a_3 \cdot A}{a_1 \cdot A}$$

and for the loss of force (B):

$$\frac{\Delta B}{B} = \frac{\sigma_B \cdot \alpha \cdot A - \tau_B \cdot b_3 \cdot B}{b_1 \cdot B}$$

Dividing the first equation by the second and assuming A=B and $a_1=b_1$ we have:

$$FER = \frac{\Delta A}{\Delta B} = \frac{\sigma_B \cdot \beta - \tau_A \cdot a_3}{\sigma_A \cdot \alpha - \tau_B \cdot b_3}$$

Granted that each force is always interested in how to improve its survivability, the only two ways to increase survivability are by increasing the defensive or the offensive power. We wish to see whether hits achieved or hits prevented is more advantageous. As we have shown, τ is a proxy for defensive power, and σ is a proxy for offensive power.

If we have weaker ships then they will have a bigger loss than the enemy. Furthermore we will show that the weak force should try to reduce its loss first. If our ships are equally strong or stronger than the opponent, our ships will first wish to ensure that they will have no loss or small loss.

We will perform two numerical examples to show that the results Lalis obtained using the computer solution with the FER does not always lead to the most operational sensible decision.

1. Equally Capable Forces

Again we assume equal numbers of ships in each force (A=B) and equal values of staying power. The two forces' ships are equally capable and they have the following characteristics:

Offensive Power: $\alpha = \beta = 3$

Defensive Power: $a_3 = b_3 = 2$

Staying Power: $a_1 = b_1 = 1$

(These are Lalis' numerical values.) We will give the same set of values for the alertness τ and scouting effectiveness σ for both forces to start with. After that, and because of symmetry, we will increase the value of A's alertness first, and the value of A's scouting second, to see what is most sensitive and the preferred attributes for A to have.

Set 1

$$\sigma_A = \sigma_B = 0.6$$

$$\tau_A = \tau_B = 0.6 \qquad FER = \frac{\Delta A}{\Delta B} = \frac{0.6\times3-0.6\times2}{0.6\times3-0.6\times2} = \frac{0.6}{0.6} = \frac{60\%\ \textit{loss of A}}{60\%\ \textit{loss of B}}$$

$$\alpha)\ \left\{\begin{array}{l} \textit{Increasing } \tau_A \textit{ to } 0.7 \\ FER = \dfrac{\Delta A}{\Delta B} = \dfrac{0.6\times3-0.7\times2}{0.6\times3-0.6\times2} \approx \dfrac{0.4}{0.6} = \dfrac{40\%\ \textit{loss of A}}{60\%\ \textit{loss of B}} \\ \textit{Increasing } \sigma_A \textit{ to } 0.7 \\ FER = \dfrac{\Delta A}{\Delta B} = \dfrac{0.6\times3-0.6\times2}{0.7\times3-0.6\times2} \approx \dfrac{0.6}{0.9} = \dfrac{60\%\ \textit{loss of A}}{90\%\ \textit{loss of B}} \end{array}\right\}$$

2. Unequally Capably Forces

We assume again equal number of units in each force (A=B) and equal value of staying power ($a_1 = b_1$), but now the ships are not equally capable. We will use the same set of values for the alertness τ and scouting σ. Since we do not have symmetry, we will observe the actions which have to be taken from each force's point of view.

The characteristics of each ship are now:

Offensive Power: $\alpha = 3$, $\beta = 4$

Defensive Power: $a_3 = 2$, $b_3 = 3$

Staying Power: $a_1 = b_1 = 1$.

(We have changed β from 3 to 4 and b_3 from 2 to 3 as Lalis did.) We will begin with the weaker force's point of view, which is A.

a. Force (A) Point Of View (The Weaker Force)

Set 2

$\sigma_A = \sigma_B = 0.6$

$$\tau_A = \tau_B = 0.6 \quad FER = = \frac{0.6\times4-0.6\times2}{0.6\times3-0.6\times3} = \frac{1.2}{0} = \frac{120\%\ \textit{loss of A}}{0\%\ \textit{loss of B}}$$

$$a)\left\{\begin{array}{l}\textit{Increasing } \tau_A \textit{ to } 0.7 \\ FER = = \dfrac{0.6\times4-0.7\times2}{0.6\times3-0.6\times3} = \dfrac{1}{0} = \dfrac{100\%\ \textit{loss of A}}{0\%\ \textit{loss of B}} \\ \textit{Increasing } \sigma_A \textit{ to } 0.7 \\ FER = = \dfrac{0.6\times4-0.6\times2}{0.7\times3-0.6\times3} = \dfrac{1.2}{0.3} = \dfrac{120\%\ \textit{loss of A}}{30\%\ \textit{loss of B}}\end{array}\right\}$$

$$b)\left\{\begin{array}{l}\textit{Increasing } \tau_A \textit{ to } 0.8 \\ FER = = \dfrac{0.6\times4-0.8\times2}{0.6\times3-0.6\times3} = \dfrac{0.8}{0} = \dfrac{80\%\ \textit{loss of A}}{0\%\ \textit{loss of B}} \\ \textit{Increasing } \sigma_A \textit{ to } 0.8 \\ FER = = \dfrac{0.6\times4-0.6\times2}{0.8\times3-0.6\times3} = \dfrac{1.2}{0.6} = \dfrac{120\%\ \textit{loss of A}}{60\%\ \textit{loss of B}}\end{array}\right\}$$

b. *Force (B) Point Of View (The Stronger Force)*

We will increase force B's alertness τ_B first and then scouting effectiveness σ_B.

Set 3

$$\sigma_A = \sigma_B = 0.6$$

$$\tau_A = \tau_B = 0.6 \quad FER = \frac{0.6\times4-0.6\times2}{0.6\times3-0.6\times3} = \frac{1.2}{0} = \frac{120\%\ loss\ of\ A}{0\%\ loss\ of\ B}$$

$$a)\left\{\begin{array}{l} Increasing\ \tau_B\ to\ 0.7 \\ FER = = \dfrac{0.6\times4-0.6\times2}{0.6\times3-0.7\times3} = \dfrac{1.2}{-0.3} = \dfrac{120\%\ loss\ of\ A}{-30\%\ loss\ of\ B} \\ Increasing\ \sigma_B\ to\ 0.7 \\ FER = = \dfrac{0.7\times4-0.6\times2}{0.6\times3-0.6\times3} = \dfrac{1.6}{0\%} = \dfrac{160\%\ loss\ of\ A}{0\%\ loss\ of\ B} \end{array}\right\}$$

Finally we will look at one more scenario to examine one of Lalis' conclusions (shown on page 25). We examine what stronger force (B) ought to do when the weaker force (A) tries to attack more effectively by increasing the value of its scouting effectiveness σ_A.

$$\sigma_A = 0.9 \quad \sigma_B = 0.7$$

$$\tau_A = 0.7 \quad \tau_B = 0.7 \quad FER = \frac{0.7\times4-0.7\times2}{0.9\times3-0.7\times3} = \frac{1.4}{0.6} = \frac{140\%\ loss\ of\ A}{60\%\ loss\ of\ B}$$

$$a)\left\{\begin{array}{l} \textit{Increasing } \tau_B \textit{ to } 0.8 \\ FER = = \dfrac{0.7\times4-0.7\times2}{0.9\times3-0.8\times3} = \dfrac{1.4}{0.3} = \dfrac{140\%\ \textit{loss of A}}{30\%\ \textit{loss of B}} \\ \textit{Increasing } \sigma_B \textit{ to } 0.8 \\ FER = = \dfrac{0.8\times4-0.7\times2}{0.9\times3-0.7\times3} = \dfrac{1.8}{0.6} = \dfrac{180\%\ \textit{loss of A}}{60\%\ \textit{loss of B}} \end{array}\right\}$$

3. Analysis of the Results

Next we examine the results we obtained in more detail, comparing them with the results Lalis obtained using the FER through a computer implementation.

In the first case of equally capable forces given that we begin with a loss of 60% in both sides, we see that by increasing alertness τ_A from 0.6 to 0.7 we reduce the loss of A from 60% to 40%. By increasing scouting effectiveness σ_A from 0.6 to 0.7 we increase the loss of B from 60% to 90%. Since the FER is changed by the same amount, the question is, which is better, to reduce the loss of A by 20% or to increase the loss of B by 30%? We claim that for this specific scenario it is better to increase A's margin of victory and survivability by almost eliminating the enemy's power to reattack through scouting effectiveness, σ_A. Furthermore, we can say that the more we increase the original values for both alertness and scouting (e.g., to 0.8 or 0.9) the more we are justified to say that it is better to increase scouting effectiveness (i.e., more effective offense). This conclusion agrees with Lalis' results (see page 25).

In the second case of unequally capable forces and from the weaker force (A) point of view we have: Given that we begin with an overkill situation of 120% for force (A), it is obvious that the weaker force (A), first of all, has to survive. For this specific set we see that (A) by increasing alertness τ_A from 0.6 to 0.7 reduces its loss from 120% to 100% (still total kill). If (A) increases scouting effectiveness σ_A from 0.6 to 0.7 instead, then (A) is able to increase (B's) loss from 0% to 30%. Taking one more step and going from 0.6 to 0.8 increment in alertness τ_A force (A) reduces its loss from 120% to 80% (still high). If (A) increases scouting σ_A 0.6 to 0.8, then (A) is able to increase (B's) loss from 0% to 60%. (We assume that (B) cannot attack first.) Thus our conclusion is that it is better for force (A) to cause great damage to force (B) (from 0% to 60%) by increasing its offensive effort (even though (A) must sacrifice itself) which they can do better by increasing scouting effectiveness σ_A.

In the third case of unequally capable forces and from the stronger force (B) point of view, we have: Given that we have a 120% loss for the weaker force (A), if force (B) tries to increase scouting effectiveness σ_B from 0.7 to 0.8, then there is no gain for (B) because we already have an overkill situation. It is as if we were to shoot the helpless enemy again.

On the other hand, if force (B) increases its alertness τ_A we again see no gain, for we only reduce the loss of (B) from 0% to -30%, which is strange. The explanation is that B has provided defensive "overkill," which is a measure of added security against an unexpected growth in A's offensive power.

From the above, we conclude that the best way for superior force (B) to improve its chances of a big victory in a missile exchange in order to increase survivability is by increasing defensive power through alertness τ_B. This conclusion agrees with Lalis' conclusion.

What follows now is the special case Lalis worked out. This is what the stronger force (B) should do when the weaker force (A) tries to increase its scouting effectiveness σ_A (tries to attack more effectively).

Given that we originally have a 140% loss for force (A) and 60% loss for force (B), then: If force (B) tries to increase its alertness τ_B from 0.7 to 0.8 then (B) reduces its loss from 60% to 30%. If force (B) tries to increase its scouting effectiveness σ_B from 0.7 to 0.8 then (B) only increases the already "overkill" of force (A) from 140% to 180%. In this case, we conclude that it is better for (B) to increase its survivability through defensive power by increasing the alertness τ_B. This conclusion disagrees with Lalis' conclusion.

V. CONCLUSIONS AND RECOMMENDATIONS

In summary, the model and results developed in chapter four apply when the following assumptions are made:

- Attrition in both forces is incurred through the application of missile salvoes exchanged by each opponents' forces.
- The results after each salvo exchange can be calculated in terms of staying power, offensive power and defensive power. These can be aggregated by using the Fractional Exchange Ratio (FER). In Hughes' words, "FER is a robust way to compare warship attributes in the absence of knowledge about the circumstances in which a warship will fight, including the attributes of the enemy." [Ref. 4]

The warship attribute of staying power is given less attention than the attributes of offensive and defensive power. Most people are interested in armament, such as the number and kind of offensive and defensive missiles a warship has. Only a few will study how many missiles a particular ship can absorb before being put out of action. It is nice if a ship can deliver its missiles as well as defend against those incoming. But, what if the ship is unable to absorb a missile and continue fighting due to a momentary failure in the ship's defense system? Once again, it is pertinent to note Hughes' admonition that, "Ship staying power's inherent robustness suggests that it should be treated with great respect." [Ref. 4]

A major purpose of this study was to examine the interrelationships between staying power and defensive or offensive power, even though staying power is more closely related to the design of the warship. This examination was accomplished in Chapter IV.

The relationship between staying power and defensive power is linear and reciprocal. The weaker the defensive power, in hits prevented, the stronger the staying power has to be, in hits that can be taken while continuing to fight.

The equation showing this relationship is:

$$a_3 = \beta - \Delta A * a_1$$

where a_3 is our warship's defensive power

β is the enemy's offensive power

ΔA is our warship's loss

a_1 is our warship's staying power

The reason for this relationship is that both attributes (staying and defensive power) work for the same final goal. They both make the effect of the enemy's missiles less powerful.

Defensive power consists of trying to intercept an enemy missile prior to impact with a friendly unit. Staying power complements the weakness of defensive power in that after a missile hits a friendly unit (because of a failure of that unit's defensive power), it is the staying power which reduces the missile's effect by enabling the warship to continue fighting.

The relationship between staying power and offensive power is non-linear and hyperbolic. The equation showing this relationship is:

$$\alpha = \frac{\Delta B}{\Delta A} \cdot \frac{(\beta - a_3)}{a_1} \cdot b_1 + b_3$$

where α is our warship's offensive power

ΔB is the enemy's loss

b_1 is the enemy's staying power

b_3 is the enemy's defensive power

The reason for this hyperbolic relationship is that offensive power is inherently and functionally different from staying power. Offensive power's function is to hit the enemy. Staying power's function is to give the opportunity for the offensive power to fire again if it fails to eliminate the enemy on the first try.

We could say that together these three attributes all work for a common goal—to increase the survivability of the warship while it completes its work on the enemy. It is important to realize that a high value of staying power contributes uniquely to survivability because the human factors of scouting effectiveness and alertness have less effect on it. The importance of a high value staying power even increases in some special situations. For example, Hughes has explained that, "In littoral operations, the effectiveness of defensive systems will be curtailed because of short response time, in which case survival and the ability

to fulfill a mission will depend more heavily on staying power." A warship which is weak in staying power relative to its offensive and defensive power is in a highly risky situation.

One cannot predict which of these three major attributes of a warship will be more preferable in every circumstance. It is impossible to track the behavior of all the warships' attributes and their interrelationships, especially when numbers of warships act together. Worthy of emphasis, however, is that staying power contributes vitally to the survivability of the warship and must be treated with greater respect by warship designers.

One last but crucial point is the advantage of numbers. This is the attribute that is the most consistently advantageous in a force-on-force engagement situation. As shown in chapter two, it is very important for a numerically smaller force to have multiple amounts of the other attributes in order to compensate for the smaller number of units.

A. RECOMMENDATIONS FOR FUTURE STUDY

- Perform a sensitivity analysis with the model to see how reasonable the model is and more importantly, what are the ranges of all parameters.
- Use different types of forces or different numbers of units to see how much the results vary.
- Try to do a validation of the model using historical or wargaming data to examine its sensitivity to the three major attributes.

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